A Direct Search Algorithm for Automated Optimum Structural Design

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The constrained optimization problem usually encountered in automated design is transformed into a single unconstrained problem through the use of penalty functions constructed so as to create a reasonably symmetrical ridge at the acceptable-unacceptable region boundary of the composite merit surface. A variable step direct search, coupled with a more extensive local search at points of direct search failure, is used to move to, and then along, this ridge. The application of this algorithm to a series of integrally stiffened cylindrical shell studies, including shells with spiral-type stiffeners, indicates that it has good convergence properties and computational efficiency.

Nomenclature

$B(D_p)$ = behavior function
$\mathbf{D}$ = vector in design variable space representing the design variables
$\mathbf{\Delta D}_p$ = exploration step in the $\hat{D}_p$ direction
D_p = components of $\mathbf{D}$, or the design variables
$\hat{D}_p$ = unit vector associated with the D_p coordinate axis
$D_p{}^L, D_p{}^U$ = lower and upper limits on the design variable D_p
e_1, e_2 = small arbitrary constants representing nearness to a constraint and finite step size used in calculating λ, respectively
e_3 = convergence test constant
J = number of behavior constraint equations
$G(D_p)$ = nondimensional constraint violation function
K_1, K_2 = step size expansion control constants
K_3 = large arbitrary constant used in Eq. (3)
$L(D_p)$ = lower limit on $B(D_p)$
$M(D_p)$ = the merit or objective function
P = number of design variables
S_p = step size magnification factor
T_p = sensitivity factor
$T_{\max}$ = largest of the T_p
$\bar{t}$ = equivalent skin thickness
$U(D_p)$ = upper limit on $B(D_p)$
$X(D_p)$ = composite merit function
$Z(D_p)$ = violation function
λ = penalty multiplier
∇ = gradient operator

Subscripts

i,j,k,p = indices

Superscripts

B = base point
r = redesign number
T = temporary base point

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Introduction

THE application of optimization theory to structural design has received considerable interest during the last two decades. Most of the earlier efforts and much of the recent effort utilize variational methods or some special concepts, such as the fully stressed design or simultaneous occurrence of stability modes. Such methods, although of theoretical and practical importance, are often difficult to apply, particularly to complex structures, or have a limited range of application. Mathematical programing methods, on the other hand, can treat a rather general formulation of the optimal design problem.[1] Furthermore, these methods permit the application of the concept of "automated design" to a broad class of engineering design problems.[2]

The constrained nonlinear optimization problem often encountered in optimum structural design is usually treated by one of two basic mathematical programing methods: constrained techniques, such as those employed by Schmit, Kicher, and Morrow,[3] Gellatly and Gallagher,[4] and Kicher[5]; and unconstrained techniques. The unconstrained methods are used in conjunction with penalty functions that transform the constrained optimization problem to an unconstrained formulation. Penalty function methods have been applied to truss design by Fox and Schmit,[6] stiffened cylindrical shells by Schmit, Morrow, and Kicher,[7] and waffle plates by Hofmeister and Felton.[8] These penalty function methods all require the solution of a sequence of unconstrained problems. All of the techniques, both constrained and unconstrained, are "interior point" methods and thus require the selection of a starting point in the acceptable region.

This paper presents an interior-exterior, single unconstrained problem, penalty function formulation using penalty multipliers given as functions of the design variables. The "direct search" of Hooke and Jeeves,[9] modified to improve its effectiveness, is utilized as the basic optimum seeking procedure. Mugele's[10] search is extended and adapted for use as an optimality check at points of direct search failure. The combined procedure is referred to as the "Direct Search Design Algorithm." This algorithm is intended as an automated design method. It eliminates the need for initial selection of a penalty multiplier, requires the solution of only one unconstrained problem, and, most important, makes the selection of an initial design more arbitrary by admitting unacceptable designs.

The elimination of the requirement for an initial acceptable design allows the designer greater freedom in his choice of a starting point. This freedom can be of significant advantage in those few cases where the problem of obtaining an acceptable design is not trivial. It also provides convenience in starting a search near a suspected optimum since optima are usually on the acceptable-unacceptable region boundary. Furthermore, where a multisynthesis path survey of the design space is required, the ability of this algorithm to use randomly selected starting points can be used to improve the effectiveness of such a survey. Most important, perhaps, is that if this optimization algorithm is used in a larger computer-aided design scheme, the problem of providing automatic or designer controlled starting points during the man-machine interplay is greatly simplified.

Weisman and Wood[11] also present an optimization method employing penalty functions and the direct search. Their procedure, however, is an interior point method that requires an initial selection of the penalty multipliers and the solution of a sequence of unconstrained problems. These multipliers are modified after each solution until a set of equality constraints is satisfied. A basic weakness of the Weisman and Wood formulation is that it ignores the problem of direct search failure. Experience with the direct search indicates that it is rather optimistic to assume that such a point is indeed a local optimum.[12,13] This problem of insuring optimality exists in all the mathematical programing methods.[2,3,5,8,12,13]

The algorithm is applied to the structural synthesis of integrally stiffened, thin, cylindrical shells under combined uniform axial compressive load and lateral pressure. Kicher[5] and Schmit, et al.,[7] treat this problem using constrained gradient and penalty function methods, respectively. Burns[14] has applied the simultaneous failure mode concept to the optimum design of such shells. In addition to the problems studied by the aforementioned authors, this paper also considers the use of internal pressure as a design variable, the inclusion of a minimum natural frequency of vibration constraint, and the design of shells with spiral-type stiffeners.

Multipath studies of the individual design problems were undertaken and the results indicate that, in this application, the algorithm appears to possess good convergence properties and seems capable of reliably locating local optima with reasonably good computational efficiency.

Direct Search Design Algorithm

The usual problem in optimal design is to maximize the merit of a design expressed as a function of the design variables within the limits imposed by a series of constraints on the design behavior (behavior constraints), and often on the size of the variables (side constraints). That is, to maximize

$$\begin{gathered} M(D_p) \qquad p = 1,2,\ldots P \\ \text{subject to the conditions that} \\ L_j(D_p) \leq B_j(D_p) \leq U_j(D_p) \qquad j = 1,2,\ldots J \\ D_p{}^L \leq D_p \leq D_p{}^U \end{gathered} \tag{1}$$

where M is the merit function, D_p the design variables, P the number of design variables, $B(D_p)$ the behavior of the design, $L(D_p)$ the lower limit and $U(D_p)$ the upper limit on behavior, J the number of behavior constraints, $D_p{}^L$ the lower limit, and $D_p{}^U$ the upper limit on the design variable D_p. These side-constraint limits can be functions of the design variables or specified constants. The merit and behavior functions can also be considered as scalar functions of the vector $\mathbf{D}$, in design variable space, having components D_p.[2,3]

In this algorithm the constraints are combined with the merit function to create the composite merit function

$$X(\mathbf{D}) = M(\mathbf{D}) - [\lambda_k Z_k]_{\max} \tag{2a}$$

if all

$$G_k \leq e_1 \tag{2b}$$

or

$$X(\mathbf{D}) = M(\mathbf{D}) - K_3 \sum_{k=1}^{2(P+J)} Z_k \tag{3a}$$

if any

$$G_k > e_1 \tag{3b}$$

where

$$\begin{aligned} G_k &= Z_k/|l_k| \\ Z_k &= \langle L_j - B_j \rangle \quad k = 1,2\ldots J \\ &= \langle B_j - U_j \rangle \quad k = J+1, J+2,\ldots 2J \\ &= \langle D_p{}^L - D_p \rangle \quad k = 2J+1, 2J+2,\ldots 2J+P \\ &= \langle D_p - D_p{}^U \rangle \quad k = 2J+P+1,\ldots 2J+2P \\ \langle Z \rangle &= \begin{cases} Z, & Z > 0 \\ 0, & Z \leq 0 \end{cases} \end{aligned} \tag{4}$$

$$\lambda_k = |2[M(\mathbf{D}) - M(\mathbf{D} + \Delta\mathbf{D}')]/[Z_k(\mathbf{D}) - Z_k(\mathbf{D} + \Delta\mathbf{D}')]| \tag{5a}$$

$$\Delta\mathbf{D}' = e_2 \nabla M(D)/|\nabla M(\mathbf{D})| \tag{5b}$$

l_k is the limit violated when $Z_k > 0$, and e_1 and e_2 are small arbitrary constants representing nearness to the constraint boundary and the finite step size in the ∇M direction, respectively. ∇ is the gradient operator and $[\lambda_k Z_k]_{\max}$ the largest of the $\lambda_k Z_k$.

When the side constraints are constants, it is usually more efficient not to include them in the composite merit function but, rather, simply to prohibit moves that violate these constraints.

The form of the penalty multiplier given in Eq. (5) limits the algorithm to problems with continuous and smooth merit and behavior functions, a restriction that is typical of all the methods previously discussed.

The composite merit function $X(\mathbf{D})$ defined by Eqs. (2) and (3) can be viewed as a surface in $P + 1$ dimensional space. Such a surface will have a sharp ridge at the acceptable unacceptable region boundary. Since the optimum design usually lies on this boundary, the general strategy is to move first to this ridge and then move along it to the optimum using a suitable ridge climbing search procedure. The direct search (also referred to as the pattern search) is such a procedure. It possesses good ridge climbing properties, does not require smoothness of the composite merit function, and is reasonably efficient at points away from a ridge. The interested reader is referred to Ref. 13 for a more thorough and illuminating discussion of the direct search algorithm.

Variable Step Size Direct Search

Preliminary experience with the direct search indicated that the search effectiveness could be drastically improved if the local exploration steps were scaled so as to produce roughly comparable changes in merit. This can be accomplished by letting the steps, $\Delta\mathbf{D}_p{}^r$, of the local exploration about the temporary base, $\mathbf{D}_p{}^{T_r}$, in the D_p direction for the rth redesign cycle be given by

$$\Delta\mathbf{D}_p{}^r = \alpha^r S_p{}^r \hat{D}_p \tag{6}$$

where

$$S_p{}^r = \begin{cases} T_{\max}{}^r/T_p{}^r, & T_{\max}{}^r/T_p{}^r \leq K_2 \\ K_2, & T_{\max}{}^r/T_p{}^r > K_2 \\ K_1, & T_p{}^r = 0 \end{cases} \tag{7}$$

$$T_p{}^r = |X[\mathbf{D}_p{}^{T(r-1)}] - X[\mathbf{D}_p{}^{T(r-1)}, + \Delta\mathbf{D}_p{}^{r-1}]/\Delta\mathbf{D}_p{}^{r-1}| \tag{8}$$

$T_{\max}{}^r$ is the largest of the $T_p{}^r$, $\mathbf{D}_p{}^{T(r-1)}$, the temporary base point for the local exploration step in the D_p direction during the previous redesign cycle, α^r the basic step size in use during the rth redesign cycle, and $\hat{D}_p$ a unit vector associated with the D_p coordinate axis in design variable space. K_1 and K_2 are arbitrarily selected constants that restrict the basic step size magnification in the event $T_p{}^r$ vanishes or becomes much smaller than $T_{\max}{}^r$. Any arbitrarily selected constants such as these will be referred to as algorithm control constants.

Experience with this modification shows that it has the additional benefit of materially improving the effectiveness of the optimality check given below.[13]

Optimality Check Procedure

Wilde[12] points out that the direct (pattern) search can fail well away from the optimum, even on relatively simple merit surfaces. The composite merit surface of a typical constrained optimization problem is likely to be much more complex than the typical unconstrained surface. This complexity can, therefore, produce frequent direct search failure. Wilde's suggestion for fitting a second, or higher-order, polynomial to the surface at such points to check them for optimality seemed unsuitable. It was felt that the presence of ridges on the composite merit surface would require the use of at least a fourth-order approximation. The effort involved in the use of such an approximation coupled with an expected high frequency of direct search failure would make such a procedure inefficient, if not impractical, in many problems. The optimality check procedure used here is, therefore, to simply perform a thorough local exploration in the neighborhood of points of direct search failure.

This optimality check local exploration procedure is adapted from Mugele's[10] search method. The last direct search base point, defined by $\mathbf{D}^{Br}$, acts as the optimality check base. The variables are initially considered two-at-a-time. Points in the quadrant defined by $\hat{D}_i$ and $\hat{D}_j$ are checked by first evaluating the merit of the point defined by

$$\mathbf{D}_{ij} = \mathbf{D}^{Br} + \mathbf{\Delta D}_{ij} \tag{9a}$$

where

$$\mathbf{\Delta D}_{ij} = (\mathbf{\Delta D}_i{}^r + \mathbf{\Delta D}_j{}^r)/2 \tag{9b}$$

If this point is inferior to the base point, then the design

$$\mathbf{D}_{ij}{}^* = \mathbf{D}^{Br} + (1 - s)\mathbf{\Delta D}_{ij} + s\mathbf{\Delta D}_j{}^r \tag{10}$$

where

$$s = [X(\mathbf{D}_i) - X(\mathbf{D}_j)]/2[2X(\mathbf{D}_{ij}) - X(\mathbf{D}_i) - X(\mathbf{D}_j)] \tag{11a}$$

$$\mathbf{D}_i = \mathbf{D}^{Br} + \mathbf{\Delta D}_i{}^r \tag{11b}$$

$$\mathbf{D}_j = \mathbf{D}^{Br} + \mathbf{\Delta D}_j{}^r \tag{11c}$$

is tested. The quantities $X(\mathbf{D}_i)$ and $X(\mathbf{D}_j)$ are always computed during direct search local exploration about $\mathbf{D}^{Br}$. If either of these points is superior to the base point, then it is used as a temporary base for the $(r + 1)$th redesign cycle using the modified direct search procedure. If both are inferior, then the three remaining quadrants associated with the coordinate axes D_i and D_j are tested. The procedure is repeated, if necessary, for all combinations of i and j, $i \neq j$.

In structural design it is often necessary to check only about half the possible quadrants, since it will be apparent in many problems that improved designs cannot exist in some of the quadrants. The amount of local exploration effort required for the direct search can similarly be reduced by not trying obviously unprotective directions.[13]

The point represented by $\mathbf{D}_{ij}{}^*$ represents the high point of a quadratic approximation to the composite merit function along a line between the points defined by $\mathbf{D}_i$ and $\mathbf{D}_j$. If this second-order check of all quadrants fails to yield an improved design, then a higher-order check can be tried. The equations for a fourth-order check sequence are given in Ref. 13 by the second of Eqs. (43) and Eqs. (45–48). In the event that a two-at-a-time search fails, the concepts previously presented can be extended to expand the search so as to consider up to P variables simultaneously with only a moderate increase in search effort.[13]

The example application showed that the frequency of direct search failure was indeed high, thus requiring the frequent application of the optimality check. It was also found that although a fourth-order check was often required to confirm optimality, the second-order check would usually locate an improved design, if one existed. Furthermore, those quadrants or combinations of variables that yielded an improved design in earlier checks were the most likely to produce improved designs in subsequent checks. It seems desirable, therefore, to perform a complete second-order check of all the combinations of design variables before employing the higher-order check, and to check first those combinations that had most recently produced improved designs. In the example application this procedure reduced the effort associated with the optimality checks to about 20% of that required for an arbitrary checking sequence.

The optimality check is applied after the direct search procedure, using a specific basic step size, fails to locate an improved design. If this check also fails to yield a superior point, then the step size is halved and a new direct search started. The process is repeated until adequate convergence is achieved or until a specified minimum step size is reached. Thus the search can be terminated if

$$\begin{gathered} |[X(\mathbf{D}^{**}) - X(\mathbf{D}^*)]/X(\mathbf{D}^{**})| < e_3 \\ \mathbf{D}^{**} \neq \mathbf{D}^* \end{gathered} \tag{12}$$

or

$$\alpha^{r+1} \leq \alpha_{\min} \tag{13}$$

where $\mathbf{D}^{**}$ is the best design located using α^r, $\mathbf{D}^*$ the best design associated with $\alpha = 2\alpha^r$, and $\alpha_{\min}$ the minimum specified basic step size. The second of Eqs. (12) is used to prevent premature termination since it was found that in some instances more than one reduction in step size was required before movement toward the optimum could be restarted.

Applications

The algorithm is applied to the minimum weight design of integrally stiffened, thin, cylindrical shells under a uniform axial compressive load and lateral pressure. A summary of the design problems treated is given in Table 1. The stiffener dimensions and spacing, and the skin thickness are treated as the design variables.

For these applications the optimization problem can be stated; find the minimum of

$$X(D_p) = \bar{t}(D_p) + [\lambda_k Z_k(D_p)]_{\max}, \quad \text{all } G_k \leq e_1 \tag{14a}$$

$$X(D_p) = \bar{t}(D_p) + K_3 \sum_{k=1}^{J+P} Z_k(D_p), \quad \text{any } G_k > e_1 \tag{14b}$$

where all

$$D_p \geq D_p{}^L \tag{14c}$$

The D_p for the conventional, ring-stringer stiffened shells and the spirally stiffened shells are defined in Figs. 1 and 2, respectively. In studies 12 and 13 the internal pressure is the design variable D_8. The symbol $\bar{t}$ represents the equivalent skin thickness. Subscript $k = 1$ is associated with local skin buckling, 2, local stringer or spiral stiffener buckling, 3, panel buckling in conventionally stiffened shells or gross spirally

Table 1 Summary of design studies

Study	Stiffener config.[a]	N_{xa}[b] p	Best $\bar{t}$	Constraints active[c]	Comments
1	inside	1,000 2	0.0491	1,2,3,4,D_4^U,D_7^U	$D_7^U = D_6^U = 0.5$, $D_p^L = 0.001$, this study is similar to Kicher's case 1. Kicher's $\bar{t} = 0.0531$.[5]
2	inside	1,000 2	0.0398	1,2,4,D_4^U	Stiffener spacing constraints relaxed, $D_7^U = 1/D_1$ and $D_6^U = 1/D_2$.
3	inside	1,000	0.0437	1,2,3,4,D_4^U	
4	outside	1,000	0.0381	1,2,3,4,D_4^U	
5	spiral outside	1,000	0.0428	1,2,3	
6	outside	0 −15	0.0528	1,2,3,4,5	$D_4^U = D_3^U = 3$ in.
7	spiral outside	0 −15	0.116	1,2,3	$D_4^U = D_3^U = 3$ in.
8	outside	10,000	0.1684	1,2,4,D_4^U	
9	spiral outside	10,000	0.1680	1,2,3	
10	outside	1,000	0.0502	1,2,4,D_4^U	The classical buckling load used in studies 1–9 and 13 is reduced by an empirical design factor in studies 10–12.
11	outside	1,000 2	0.0393	1,2,3,4,D_4^U	Less conservative load reduction factor used with pressurized shells.
12	outside	1,000 21	0.0250	1,2,4,5,D_4^L	Internal pressure considered a design variable. Monocoque $\bar{t} = 0.0278$, $p = 32$ psi. $D_p^L = 0.001$, $p = 1,2\ldots5$.
13	outside	1,000 22	0.0229	1,2,4,5,D_4^L	Similar to 12 except a minimum natural frequency of vibration constraint of 20 rad/sec is imposed in lieu of a gross buckling constraint.

[a] Conventional stiffeners unless otherwise specified.
[b] N_{xa} is the applied uniform axial compressive load in lb/in. and, p, the lateral pressure in psi with internal pressure taken as positive.
[c] The lower and upper limit side constraint constants are 0.01 in. and 1 in., respectively, unless indicated otherwise.

stiffened shell buckling, 4, conventionally stiffened shell gross buckling or minimum natural frequency of vibration (study 13), and 5, ring buckling (study 6) or skin yield (studies 12 and 13). The upper side constraint limits are included in $X(D_p)$ since most of them are not constants, thus $k = J + 1, J + 2, \ldots J + P$ are associated with $D_1^U, D_2^U \ldots D_p^U$, respectively, where $J = 4$ in design studies 1–4 and 8–11, $J = 3$ in studies 5, 7 and 9, and $J = 5$ in studies 6, 12, and 13.

For a conventionally stiffened shell

$$\bar{t} = D_5 + |D_1D_3D_7| + |D_2D_4D_6| - \langle D_1D_2D_3D_4D_6D_7/D_m\rangle \quad (15)$$

where the meaning of the brackets of the last term of Eq. (15) is the same as in Eq. (4). A negative sign is associated with D_3 and D_4 for interior stringers or rings, and D_m is the smaller of $|D_3|$ and $|D_4|$. Here D_p^L and D_p^U are taken as arbitrary constants except $D_1^U = 1/D_7$, $D_2^U = 1/D_6$, $D_6^U = 1/D_2$, and $D_7^U = 1/D_1$.

For shells with paired, 45°, spiral-type stiffeners

$$\bar{t} = D_1 + 2D_2D_3D_4 - (D_2D_4)^2D_3 \quad (16)$$

This study of spirally stiffened shells is limited to cases where both sets of stiffeners are on the same side of the shell. D_3, therefore, does not require a sense designation and is always considered as positive. Here again the side constraint limits are constants except that $D_2^U = 1/D_4$ and $D_4^U = 1/D_2$.

Equations for evaluating the Z_k are given in Appendix A of Ref. 13. The local skin and stiffener buckling equations are similar to those used by Schmit et al.[3] except that the dimensions of the local skin buckling plate element are assumed to be equal to the stiffener spacing, rather than the stiffener spacing less the stiffener thickness. A classical buckling load value (N_{cl}) is used in studies 1–9 and 13 for the panel and gross buckling constraints. Studies 10–12 employ a reduced buckling load, $N_{cr} = \phi N_{cl}$, where the load reduction factor ϕ is based on experiments with unstiffened shells. Reference 15 indicates (refer particularly to Fig. 2 of Ref. 15) that N_{cr} can be considered to provide realistic design values. The equations for N_{cl} and ϕ used in the conventionally stiffened shell studies are taken from Burns[14] and N_{cl} for spirally stiffened shells is given by Soong.[16]

Results and Discussion

The algorithm control constants and design parameters used in these studies are, $e_1 = 0.1$, $e_2 = 0.001$, $e_3 = 0.01$, $K_1 = K_2 = 10$ except in studies 12 and 13 where $K_1 = D_8/D_5$, $\alpha^0 = -0.005$ in., $\alpha_{\min} = -0.0001$ in., and $K_3 = 10^4$. The constants associated with D_p^L and D_p^U are 0.01 in. and 1 in., respectively, except in design studies 1, 6, 12, and 13 where $D_p^L = 0.001$, $p = 1,2\ldots5$, and in studies 12 and 13 where $D_8^L = 0$ psi and $D_8^U = 100$ psi. $R = 60$ in., $L = 165$ in., Young's modulus $= 10^7$ psi, Poisson's ratio $= 0.333$, and the yield strength is taken as 72 ksi. Variables D_1–D_7 and $\bar{t}$ are given in inches and D_8 in psi. The optimality check employs a two-at-a-time quadratic search of all feasible quadrants followed by a similar fourth-order check. The historically ordered checking sequence discussed earlier is used. Most runs were terminated after the step size was reduced below $\alpha_{\min}$ so that the effect of step size reduction on convergence could be studied. The convergence test was, however, used in some instances. A Burroughs B5500 computer was used for all studies.

Preliminary investigations involving design study 4 showed that an equal step direct search was ineffective in this application. A synthesis run using the equal step search was terminated manually after 1145 redesign cycles. During this run the quadratic check was called 251 times and the fourth-order check 42 times. The best design obtained had a $\bar{t}$ = 0.0563. A variable step search would usually achieve such values of $\bar{t}$ within 30 redesign cycles. Preliminary runs where the optimality check was used only after the search step size was reduced to a specified minimum demonstrated that the direct search must indeed be augmented by an optimality check for the composite merit function given here.

Table 2 illustrates the convergence properties of the algorithm. The table demonstrates that once a reasonable degree of convergence had been achieved additional step size reductions produce little improvement in merit. This data is taken from the results of the five synthesis path survey, using widely separated starting points, referred to in Table 3. Such multipath surveys were also made for design studies 4, 5, 12, and 13. It may be seen that the algorithm has excellent convergence properties in this application. All the runs converged to the same stiffened (paths 1, 2, and 3) or unstiffened (paths 4 and 5) local optimum. Table 3 also illustrates that although the algorithm admits an arbitrary starting point, an extremely poor choice may result in convergence to the local unstiffened optimum. These results were typical for all design studies where multipath surveys were made except for study 13. Here two essentially distinct stiffened designs were located having a weight difference of about five percent.

The number of redesign cycles required to achieve a convergence of 1%, and the computer running times, given in Table 3, can also be considered typical for design studies 1–4, 8, 12, and 13. In the case of hydrostatic pressure (study 6), a ring-stiffened pseudo-optimum slowed progress considerably. The studies of the spirally stiffened shells required much less effect and would achieve convergence in about 30 redesign cycles and 2-min processor time.

The computer running times of the conventionally stiffened shell design studies can probably be reduced about 40% by writing the equations given by Burns[14] in the form used by Soong.[16] This would allow a more efficient search for the minimum buckling load since the great majority of the terms, those which are functions of the design variables only, would have to be calculated only once during this search procedure.

It is rather difficult to draw reasonably reliable quantitative comparisons between the direct search design algorithm and the methods used in Refs. 5 and 7 because of the detail differences in the treatment of constraints. Furthermore, even though the computers used in all three studies are somewhat similar in performance, the differences in the computing machinery and programing efficiency compound this difficulty. The comparison of running times, convergence characteristics, and final designs produced, nevertheless, suggests that the direct search is roughly comparable in effectiveness to the sequential, unconstrained minimization technique of Ref. 7 and therefore superior to the constrained gradient method used by Kicher.[5]

Table 2 Effect of step size on the best value of $\tilde{t}$

	Best $\bar{t}$		
$\alpha \times 10^{-3}$	Path 1	Path 2	Path 3
5	0.052142	0.050820	0.050754
2.5	0.051164	0.050530	0.050677
1.25	0.050682	0.050474	0.050288
0.625	0.050469	0.050396	0.050246
0.312	0.050422	0.050351	0.050242
0.156	0.050414	0.050347	0.050234
0.078	[a]	[a]	0.050224
0.039	0.050412	0.050344	0.050221
0.019	[a]	0.150343	[a]

[a] No significant change.

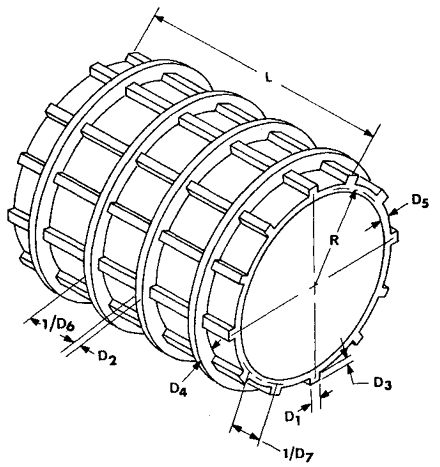

Fig. 1 Design variables and dimensions for shells with conventional stiffeners.

A typical synthesis path is given in Table 4. It can be seen that direct search failure occurred early and rather frequently. A total of 26-second-order and 5-fourth-order checks were called before the step size was reduced. The use of the fourth-order check resulted in a relatively minor reduction in $\tilde{p}$ for this run. Although this situation is typical for most runs, it was found that in general the higher-order sequence was required to provide a reasonable degree of confidence in the design.

Initial synthesis runs for the design studies involving shells under hydrostatic pressure (studies 6 and 7), using a maximum stiffener height of 1 in., produced a ring-stiffened conventional design with $\bar{t}$ = 0.148 and a spirally stiffened design with $\bar{t}$ = 0.174. In both instances the stiffener height variable converged to the maximum. The relaxation of this constraint produced drastically improved designs (refer to Table 5). In design study 2, the relaxation of the minimum stiffener spacing constraints of 2 in. used in study 1 similarly resulted in a significant weight reduction.

Studies 12 and 13, where internal pressure is used as a stiffening element, and also used to support part of the applied axial load, produced stringer-stiffened designs and a somewhat heavier monocoque local optimum. These stiffened designs use much lower values of internal pressure. They can, therefore, sustain appreciably greater loads than the monocoque design in the event of accidental depressurization and would require lighter weight pressurizing equipment.

An examination of synthesis paths of these two studies showed that there exist several, if not many, nearly equal weight, and yet significantly different, stringer-stiffened designs. The two synthesis paths of study 12 that yielded a stiffened configuration produced essentially similar but still

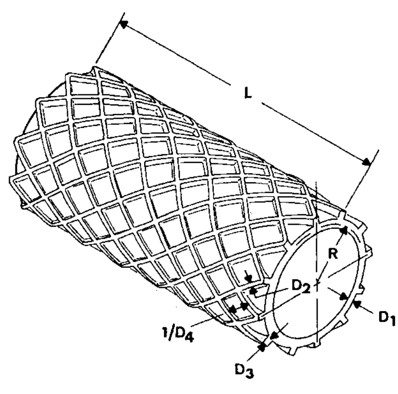

Fig. 2 Design variables and dimensions for shells with spiral-type stiffeners.

Table 3 Multisynthesis path survey of design study 10

	Path 1	Path 2	Path 3	Path 4	Path 5
Initial design					
D_1	0.5	0.1	0.02	0.01	1
D_2	0.5	0.1	0.02	0.01	1
D_3	1	1	0.5	0.01	1
D_4	1	1	0.5	0.01	1
D_5	0.5	0.05	0.05	0.01	1
D_6	0.01	0.1667	0.5	0.01	1
D_7	0.01	0.333	0.5	0.01	1
Best design					
D_1	0.0379	0.0378	0.03819	0.01000	0.01000
D_2	0.03681	0.0271	0.03575	0.01000	0.01000
D_3	0.5474	0.549	0.5574	0.01000	0.01000
D_4	1.000	1.000	1.000	0.01000	0.02445
D_5	0.02422	0.267	0.02674	0.18474	0.18474
D_6	0.1403	0.179	0.1245	0.01000	0.01001
D_7	1.010	0.911	0.9077	0.06548	0.06547
Best $\bar{t}$	0.05022	0.05034	0.05041	0.18474	0.18475
$\bar{t}^a$	0.05068	0.05053	0.05068	0.18474	0.18480
Time[a] min	9.70	6.35	8.95	3.53	2.38
r^a	105	74	70	26	37
$\alpha \times 10^{-3a}$	2.5	2.5	1.25	0.0781	1.25

[a] At convergence, $\epsilon_3 = 0.01$.

Table 4 Typical synthesis path for design study 4

Redesign no.	$D_1 \times 10^{-1}$	$D_2 \times 10^{-1}$	D_3	D_4	$D_5 \times 10^{-1}$	D_6	D_7	Constraints active	$\bar{t} \times 10^{-1}$	$\alpha \times 10^{-3}$	2nd[a]	4th[b]	Comments
0	1	1	1	1	0.5	0.1667	0.333	1	—	5	—	—	$G_1 = 0.168$
7	1.143	0.100	0.855	0.739	0.500	0.0517	0.343	3,D_2^L	0.878	5	1	—	0.39 min
16	0.534	0.100	0.630	0.835	0.625	0.0517	0.328	1,3,D_2^L	0.740	5	2		
33	0.330	0.333	0.334	0.727	0.450	0.152	0.578	3,4	0.550	5	7		
56	0.305	0.333	0.352	0.877	0.225	0.152	1.270	1,3	0.405	5	15	—	3.22 min
62	0.305	0.333	0.377	0.766	0.225	0.110	1.220	1,3	0.392	5	19	1	3.80 min
69	0.292	0.333	0.372	0.894	0.214	0.110	1.304	2,3,4	0.388	5	26	5	Reduced step size to 0.0025, 5.21 min
72	0.292	0.333	0.373	0.859	0.214	0.112	1.292	1,2,3,4	0.386	2.5	3	2	Met convergence test, 6.28 min

[a] Number of times the second-order optimality check was called at the particular step size in use at this redesign number.
[b] Number of times the fourth-order optimality check was called at the particular step size in use at this redesign number.

significantly different designs: the first (D_1 = 0.01825, D_3 = 0.4228, D_5 = 0.02322, D_7 = 0.3475, and D_8 = 26.51) with a $\bar{t}$ = 0.02510 and the second (D_1 = 0.02595, D_3 = 0.04596, D_5 = 0.01986, D_7 = 0.4310, and D_8 = 21.04) with $\bar{t}$ = 0.02503. The three synthesis paths producing stiffened designs in study 13 also yielded designs with distinct differences. Two are moderate pressure, medium thickness skin designs, the first (D_1 = 0.01629, D_3 = 0.2872, D_5 = 0.02100, D_7 = 0.4050, and D_8 = 22.28) with $\bar{t}$ = 0.02290 and the other (D_1 = 0.01242, D_3 = 0.02728, D_5 = 0.02308, D_7 = 0.2300, and D_8 = 25.72) with $\bar{t}$ = 0.02390. The third is a relatively low pressure, thin skin design (D_1 = 0.0262, D_3 = 0.3700, D_5 = 0.0145, D_7 = 0.9882, and D_8 = 13.62) with $\bar{t}$ = 0.02377. These results suggest that either the composite merit surfaces for these studies have several, and perhaps many, local optima or that the two-at-a-time optimality check could not cope with the increased complexity of these surfaces. The location of the two rather distinct design concepts in study 13, and the nature of the differences in the various designs, indicate that the former is likely to be the principal reason for the lack of convergence to a single stiffened design. The subject, however, warrants further investigation.

Table 5 Summary of optimum design dimensions

Study	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	$\bar{t}$
1	0.0258	0.0082	−0.395	−0.999	0.04267	0.164	0.500	—	0.0491
2	0.0297	0.0101	−0.399	−1.000	0.02520	0.296	0.966	—	0.0398
3	0.0321	0.0329	−0.436	−1.000	0.02604	0.114	0.998	—	0.0437
4	0.0297	0.0204	0.382	1.000	0.02070	0.110	1.336	—	0.0381
5	0.02333	0.0211	0.369	1.274	—	—	—	—	0.0428
6	0.0101	0.135	0.190	2.144	0.03262	0.0580	1.359	—	0.0520
7	0.05238	0.0560	1.397	0.399	—	—	—	—	0.116
8	0.117	0.217	0.948	1.000	0.08996	0.123	0.483	—	0.168
9	0.09097	0.0829	0.910	0.522	—	—	—	—	0.168
10	0.0379	0.368	0.547	1.000	0.02422	0.140	1.010	—	0.0502
11	0.0555	0.0343	0.747	1.000	0.02054	0.0705	0.395	—	0.0393
12	0.0260	0.0961	0.460	0.001	0.01986	0.246	0.431	21.04	0.0250
13	0.0163	0.0418	0.287	0.001	0.02100	0.0194	0.405	22.28	0.0229

Conclusions

The Direct Search Design Algorithm appears to be an effective tool for automated structural design. It seems to possess good convergence properties and reasonably good computational efficiency. It has the important advantage of allowing the use of an arbitrary starting point.

A two-at-a-time optimality check was adequate for all design studies except perhaps those where internal pressure was treated as a design variable. Since the effectiveness of this check can be materially improved by extending the number of variables checked simultaneously, it can be concluded that, using this extension, the algorithm should be capable of seeking out local optima with comparatively good reliability. The multisynthesis path surveys undertaken for most of the design studies indicate that again with the exception of studies 12 and 13 the best designs located are probably optima. In studies 12 and 13 it is likely that the best designs are at least near optima.

The designer should be aware, however, that the general nonlinear constrained optimization problem is quite formidable, particularly for large order systems, and none of the available techniques, including this one, can guarantee a feasible solution. An automated design capability must be demonstrated for a given problem type before it can be assumed to exist. Such capability apparently does exist for the structures studied here.[7]

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